

An introduction to elliptic operators: a brief description and syllabus

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In this course we will cover the basics of the theory of elliptic operators, examples of which include the de Rham and Dolbeault operators, as well as their generalisations such as the Dirac operator. We will explain how several seemingly unrelated results in geometry and topology (e.g. the Hirzebruch and Rokhlin signature theorems and the Riemann-Roch theorem) all follow from the general index formula by M. Atiyah and I. Singer, and sketch a proof of the latter. The main prerequisites are smooth manifolds and singular cohomology. We will recall some or all of these if necessary. Here is a tentative syllabus.

- Vector bundles and characteristic classes: a summary of results.
- Differential operators: the definition and first examples.
- Elliptic operators: the definition and basic properties.
- Riemannian metrics on manifolds and the de Rham operator.
- The signature operator.
- Complex manifolds and the Dolbeault operator.
- Clifford algebras and their representations.
- Reduction of the structure group of a vector bundle. Spin structures on vector bundles.
- Dirac operators. Constructing Dirac operators using Spin structures.
- Elliptic regularity and related results about elliptic operators on compact manifolds.
- First applications: the de Rham and Hodge decomposition theorems; the Serre duality.
- The Atiyah-Singer index formula.
- Applications of the index formula: the Riemann-Roch theorem, the Hirzebruch signature theorem, V. Rokhlin's signature theorem.

The main references are

- *Algebraic Topology* by A. Hatcher, freely available online at <http://www.math.cornell.edu/~hatcher/AT/ATpage.html>.
- *Spin geometry* by H.B. Lawson and M.-L. Michelsohn.
- *Seminar on the Atiyah-Singer index theorem* by R.S. Palais et al.
- *Characteristic classes* by J. Milnor and J. Stasheff.
- *The Atiyah-Singer index theorem* by P. Shanahan.
- *Differential analysis on complex manifolds* by R.O. Wells.

Occasionally we'll be using other sources as well.